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Determination of earthquake focal mechanism
Description of six computer programs

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Summary.

In recent years six computerprograms have been developed by several investigators in order to determine the focal mechanism of an earthquake. These programs are rewritten for the Burroughs B6700.
In chapter 2 the programs are described briefly, especially for the users of the programs.
In chapter 3 some rules are given for the choice of the best program.
For references see chapter 4.

Determination of earthquake focal mechanism.

1. Introduction.

The mechanism in the focus of an earthquake is determined by the direction of the first motion of longitudinal and transverse waves as recorded at seismic stations in many directions and distances around the focus. Such data, reprojected down on the focal sphere always show a distinct pattern: The first motions of longitudinal (P-) waves are distributed in compression and dilatation quadrants that are separated according to two orthogonal nodal planes along which amplitudes are reduced to zero. The directions of polarization of transverse (S-) waves follow certain patterns that are closely related to the position of the nodal planes in the P-wave radiation.

The position of the nodal planes in the P-wave radiation, and the characteristic points in the S-wave radiation are indicative of the relative movements occurring in the focus of the earthquake.

The study of focal mechanism has long been restricted to that of longitudinal wave radiation, since this is the most clear information that seismograms can offer, being that of the first arriving wave. But if the quality of the data is adequate, also the transverse wave radiation can be used to the same effect.

Until 1963 the fault-plane solutions for earthquakes using longitudinal waves had only been obtained graphically with the subjective trial-and-error method. Generally this method leads to satisfactory results. Nevertheless, it always remains uncertain whether the solution obtained in this way is the absolutely best one or not. Sometimes indeed, the results of investigations by different authors for one and the same earthquake are different.

If a solution could be obtained by computation through purely mathematical procedures one would have more confidence in the result because of the objectivity of the produced solutions.

In 1941 S. Homma (1) proposed the application of the least squares method for solving the fault-plane problem using P-waves only. Unfortunately, his scheme was not successful.

In 1961 Knopoff (2) and (3) considered the enhanced probability for a station situated near one of the nodal planes to report an incorrect first motion. To this effect he introduced a kind of probability function, which he then maximized for the determination of the optimal location of the two orthogonal planes.

Kasahara (4), in 1963, improved this method. A computer program was written producing the optimal solution by a series of successive approximations from a given first assumed position of the nodal planes for longitudinal waves.

In 1965 the most widely used program of Wickens and Hodgson (8 and 9) was written. No initial first approximation of the position of the nodal planes is needed. By an extensive systematic search the program determines the optimal position of the P nodal planes and the B-axis for any set of P-wave observations. Moreover, the possible variation in these positions is calculated.

In 1964, Stevens (11 and 12) was the first to write a program using S wave data. By solving a family of equations depending only on S wave polarization angles the orientation of the force system causing the earthquake is determined. The position of this force system is interrelated with that of the nodal planes for longitudinal waves to be determined by one of the earlier computer programs.

In 1966 another program was written by Hirasawa (13 and 14) to determine the focal mechanism by S-wave data only. In this program the sum of squares of the residuals of the polarization angles of S waves is minimized.

A computer program to find the orientation of the stress axes and nodal planes of an earthquake satisfying optimally the data from both P and S waves was developed by Udias and Baumann (17) in 1969. This work is based mainly on an earlier program of Udias (16) for the use of S waves only.

Still another program for the determination of the earthquake focal mechanism, combining both P and S wave data, was written by Chandra (19) in 1971.

Now these six programs are evaluated and re-written for the Burroughs B6700 computer. In chapter 2 the programs are described in more detail (method, input and output), and in chapter 3 some conclusions are drawn.
It is now possible to compare these programs for several applications. In a next publication these applications will be discussed.

2. Program description.

2.1. Kasahara (P-wave data only).

The method of Kasahara is described in (4).
The maximum of a probability function defined by Knopoff gives the most probable solution. From a given first approximation the final solution is obtained by a series of successive approximations.
The program is written in FORTRAN and processes data up to 190 stations.
In general, a time of 2 to 10 seconds is required for a solution involving 20 stations (for the Burroughs B6700).

2.1.1. Input.

Coordinates to be used.

Draw a focal sphere and take the X-, Y- and Z-axes southwards, eastwards and upwards, respectively.
Draw the two nodal planes on the basis of the first-approximation solution.
Take the y-axis as the upward extension of the null vector.
Take the x- and z-axes in the two nodal planes, so as to form the right-hand system in the upper hemisphere.
Name the plane involving (y,z) as plane A and another (y,x) as B. Name the intersection of the equator for the Z-axis with the planes A and B as P_A, P'_A; P_B, P'_B, respectively, where (') should be used so that the following relations may be satisfied,

$$yP_A \leqslant yP'_A \quad , \quad yP_B \leqslant yP'_B .$$

Input data.

Input data consist of three groups of cards as follows:

a. directions for computation
b. station data
c. first approximation solution.

a. Directions for computation should be given on a single card with the format (6X, I4, 2(5X, F5.2), 6X,F4.1) in the following way:

N, ZD, ZE, POL,

where N: total number of station data (max. 190 stations)

ZD and ZE: perturbation parameters for computing probable errors of solution (ZD = 0.20 and ZE = 1.00 would be proper

POL: polarity of forces at the origin (POL = + 1.0 compression or - 1.0 dilatation).

b. Station data consist of cards as many as the stations to be used for analysis. Each card should contain the following data:

IX1, FI, THTA, IX2,

where IX1: identification number of the station

FI : azimuth to the station measured clock-wise from north (degrees)

THTA: emergent angle of seismic ray to the station measured from the direction ZO (degrees)

IX2: polarity of the initial P-pulse (sign) and weight of the data (numeral 1 until 9)

with the format (6X, I4, 2(3X, F7.2), 8X, I2).

c. Solution of the first approximation should be given on a single card as follows:

Z(1), Z(2), Z(3), Z(4),

where Z(1): $\angle XOP_A$ (not XOP'_A) measured counter clock-wise from OX

Z(2): $\angle XOP_B$ measured counter clock-wise from OX

Z(3): dip of the plane A

Z(4): β in (4) (β = 0.10 would be proper).

with the format (4(3X,F7.2)).

2.1.2. Output.

The program prints out:

a. The successive approximations of Z(1), Z(2), Z(3), Z(4) and the value of the probability function.

b. Identification number of stations reporting inconsistent sign with polarity and theoretical amplitude.

c. The final values of:

- Z(1), Z(2), Z(3) and D_B with their standard errors.

Z(1) = the strike of plane A

Z(2) = the strike of plane B

$Z(3)$ = the dip of plane A

D_B = the dip of plane B

- Azimuth and plunge of the null vector, β and the probability function.

2.2. Wickens. (P-wave data only)

The method of Wickens and Hodgson is described in (9).
An orthogonal frame defining the fault plane and the auxiliary plane or the nodal planes separating regions of compressions and dilatations is systematically rotated at the hypocentre so as to obtain the fit to the data for practically all possible orientations. A spacing of delta azimuth = 10°, delta plunge = 10° and delta rotation on null = 6° (i.e. 4860 trials) is fine enough to avoid overlooking a solution. A specified number - 20 - of the best trials from the 4860 are retained for refinement over a finer subgrid.
Finally, the best refined solution is kept as well as the solution with the least number of inconsistent stations, if it is different from the best. This is possible, since the stations are weighted independently. The program is written in FORTRAN and processes data up to 200 stations. The execution-time is about 1.5 minutes for a solution involving 20 stations and about 9 minutes for 130 stations.

2.2.1. Input.

The input data consist of:

a. Tables of extended distance for P, PKP1 and PKP2-waves. Table size leads each table. A format of (8F8.3) is used for size (word one) and tables. (Word one on each table card is the angular distance).

b. Symbol table, format (20A1, 3A3).

c. Control card for delta azimuth (A), delta plunge (H), delta rotation on null (R), terminal (fine grid), convergent factor for (fine grid), minimum value for theoretical weight (step), number of solutions to be retained for refinement from initial trials (The values 10°, 10°, 6°, 0.1, 0.5, 0.3 resp. are very satisfactory from the point of view of reliability and efficiency). Format (8F8.3).

N.B. a, b and c are normally fixed for all solutions.

d. Epicentre (latitude and longitude), date, H-time, depth and extended distance (which specifies limit of plot). Format (2(F10.3, A2), 4F10.1).
   If blank card then program terminates.
e. Station 3 letter code, azimuth (epicentre to station), angular distance, phase (P, PKP1 or PKP2, where P, 1 and 2 are in same column), polarity (D or C), weight (100 if reliable, 1 if unreliable). Format (A4, 2F10.3, 4X, 2(3X, A1), F6.0).
f. Blank card switches to new epicentre card.

2.2.2. Output.

The program prints out:

a. Same as input d. (the extended distance for plot limit is omitted). Data are in days, months and year. H-time is in hours, minutes and seconds.
b. Table giving main results:

   Line one:

   Score, total number of observing stations, number inconsistent stations, number of inconsistent stations which were weighted as unreliable (i.e. weight = 1), azimuth of dip, dip, strike slip and dip slip components for each plane (S for sinestral, D for dextral, N for normal and T for thrust), azimuth and plunge (pressure, null, tension axes).

   Line two:

   Facts from line one after a further refining process with the inconsistent observations given a weight 2 (effectively zero). Stations which are initially weight zero (i.e. wt = 1) are not set at weight 2.

   Line three to eight:

   First column indicates angular rotation of orthogonal frame about each of the axes in turn, sufficient to make two additional observations inconsistent. The remainder of the information indicates the corresponding ranges of azimuths and dips for the above rotation limits.

   Line nine:

   Cones as defined by A.R. Ritsema in a paper titled "Some Reliable Fault Plane Solutions", Pure and Applied Geophys., v.59, 58-74, Basel, 1964/III.

Line ten:

Direction cosinus of each of the axes.

c. Three letter station code, plot coordinates, "W" to indicate inconsistent observations, normalized theoretical amplitude at the hypocentre, observed polarity assigned to weight (1, 2 or 100), azimuth epicentre to station, emergent angle (measured up from downward vertical) at hypocentre, angular distance delta, word "OFF" will appear after delta if station is beyond the limits of the plot, station direction cosines for tangents to ray path at hypocentre.

d. Byerley (stereographic) plot of data with coordinates of circle centre and extended distance defining plot limit on first line. Coordinates are in terms of "period" spacing that delineates the axis. The plot code is as follows: L (3 neg.), M(2 neg.), N(1 neg.), O(1 neg., 1 pos.), P (1 pos.), Q (2 pos.), R (3 pos.) and J any other combination.

e. Stereographic projection.

2.3. <u>Stevens.</u> (S-wave data only)

The method of Stevens in described in (11).

A family of equations is derived which depends only on S polarization angles and not on initial P displacements to determine the orientation of the force system at the foci of earthquakes.

The program is written in FORTRAN and processes data up to 50 stations.

The execution-time is about 1 minute for a solution involving 20 stations.

2.3.1. <u>Input.</u>

The input data consist of:

a. Station data.

For one station we used one card. Each card contains the following data:

reference number of the station (NO), azimuth (PHI), emergent angle (THETA), polarization angle (ETA), date (ND, NM, NY) and horizontal polarization angle (GAMMA).

b. card after last station with NY = 0.
c. end of job card with NY = 999.
The format of a, b and c is (5X, I5, 3X, F7.2, 4X, F6.2, 3X, F7.2, 1X, 3I3, 14X, F6.1).

2.3.2. Output.

The output needs no further description, it is clear when you see the computer prints.
The main results are azimuth of dip and dip for each plane and azimuth and plunge for pressure, null and tension axes.

2.4. Hirasawa. (S-wave data only)

The method of Hirasawa is described in (13) and (14).
It is a least-squares-method for the focal mechanism determination. The method is based upon the assumption that a finite set of the residuals of the polarization angles of S waves for a particular earthquake is sampled from a normal distribution, where the residual is defined as the deviation of the observed polarization angle from that expected theoretically at the particular (individual) station. The sum of squares of deviations (residuals) is minimized.
Because of the non-linearity of the problem the sum of squares of the residuals in some cases has secondary minima.
In practice many trial solutions are needed to find all minima. These are obtained from several sets of three observations. Although any combination of three observations taken arbitrarily from the available observations can be used, it may be better to avoid the use of the combination of observations from stations located close to each other. The sum of squares of the residuals is computed for each of these trial solutions. Several solutions are selected from these trial solutions for the starting points of the parameters in the iterative process of the least squares. Finally, the fault-plane solution is calculated.
The program is written in ALGOL and consists of three parts:
- part one for the first approximate solution,
- part two for the successive approximations of least squares and

- part three for the final values (the fault-plane solution and the standard deviations).

The program processes data up to 60 stations.
The execution-time is about 15 seconds for a solution involving 20 stations, but this depends strongly on the number of trial solutions.

2.4.1. Input (part one).

The input data consist of:

a. The numerical order of the earthquake in years, months and days, if this is zero then STOP, Format (I10).
   STOP means the computer-program comes to an end.
b. The number of observations. Format (I3).
c. For each station in degrees: the emergent angle, azimuth and horizontal polarization angle. Format (3(F7.2, X3)).
d. For each trial solution:
   - the numerical order of the set of the three observations, if this is zero then START. Format (I2).
     and
   - the numerical order of three observations in the set. Format (3(I3, X2)).
     START means the program goes to the beginning.
e. Two blank cards.

2.4.2. Output (part one).

The main results are AC=X1, BC=X2, DC=X3, EC=X4 and RES (sum of squares of the residuals). For each trial solution we find one or more of these values. Several solutions are now selected with the criterion that the sum of squares of residuals is minimized.

2.4.3. Input (part two).

The input data consist of:

a. The same as part one.
b. The same as part one.
c. The same as part one.
d. For each selected solution (output part one):

- the numerical order of the solution, if this is zero then START.
  Format (I2)
  and
- the starting values of the solution (X1, X2, X3).
  Format (3(F15.10, X5)).

e. Two blank cards.

2.4.4. Output (part two).

The main results for each selected solution are:

- X1, X2, X3 and X4
  and
- the strike, dip direction and inclination angle of the two nodal planes.

2.4.5. Input (part three).

The input consist of:

a, b and c the same as part two.

d. For each least-squares-solution (output of part two):

- the numerical order of the solution, if this is zero then START. Format (I2)
  and
- the starting values of the solution X'_1, X'_2, X'_3 with $X'_1 = X_1$, $X'_2 = X_2$ and $X'_3 = 1/X_4$. Format (3(F15.10, X5)).

e. Two blank cards.

2.4.6. Output (part three).

For each least-squares-solution the main prints are:

- the final values of X'_1, X'_2, X'_3 and X'_4,
- the dip angle, dip direction and slip with their standard deviations of the two nodal planes
- and (90^o-plunge), azimuth with the standard deviations of the null axis.

2.5. Udias and Baumann. (both P- and S-wave data).

The method of Udias and Baumann is described in (17) and (18). A computer program has been written to determine the focal mechanism combining P- and S-wave data.

The program, therefore, uses a number of observations with the sign of P and the horizontal polarization angle of S.
The program takes as a starting point the solution on S-data and then imposes the condition that the P-data must also be satisfied. The main lines of the method which uses S-wave data follow the previous work of Udias (16) with same modifications.

The method has been programmed in FORTRAN and processes data up to 100 stations.
The execution-time is about 1 minute for a solution involving 20 P observations and 20 S observations.

2.5.1. Input.

The input data consist of:

a. The values of angles of incidence of P-waves for the values of the distance 1 to 140 degrees (see (15)). Format (10F8.2).
b. The same for the S-waves.
c. The identification code of the earthquake, coordinates of the earthquake (latitude and longitude), date (day, month, year), H-time (hours, minutes, seconds) and depth. Format (A4, 2F9.3, 8X, I2, A3, I4, 4X, 2I2, 2F8.1).
d. The factor D. If D = 0 then the solution is based on only S-waves, if D = 100 then the solution is based on a combination of P and S waves. The constant CODE (CODE = 1). Format (2F10.5).
e. The first approximation of Θ_x and Φ_x (initial orientation of X - axis) where Φ_x is measured from north c.c.w. and Θ_x from the downward vertical. Format (2F10.5).
f. For each station (max. 100): the identification code of station, latitude and longitude. Format (A4, 2F9.3).
g. A card with latitude is 100.
h. For each P-observation (max. 100): the identification code of the station and the sign of P (+1 or -1). Format (A4, F9.3).
i. A card with sign of P is 2.

j. For each S-observation (max. 40):
   the identification code of the station, the horizontal polarization angle of S and the estimation of the possible error of the horizontal polarization angle. Format (A4, 2 F9. 3).

k. A card with the polarization angle is 500.

2.5.2. <u>Output.</u>

The computer prints need no further description.
The main prints are the trend and plunge of the X, Y, pressure, null and tension axes.

2.6. <u>Chandra.</u> (both P- and S-wave data).

The method of Chandra is described in (19).
A method has been proposed for the combination of P-wave first-motion directions and S-wave polarization data for the numerical determination of earthquake focal mechanism. The method takes into account the influence of nearness of stations with inconsistent P-wave polarity observations, with respect to the assumed nodal planes and gives a reasonable weight to both P- and S-wave data.

The method has been programmed in FORTRAN and processes data up to 99 stations.
The execution-time is about 20 seconds for a solution involving 20 P-observations and 20 S-observations.

2.6.1. <u>Input.</u>

The input data consist of:

a. the coordinates of the epicentre (latitude and longitude), date (days, months, years), origin time (hours, minutes, seconds), depth, number of P-observations, number of S-polarization observations and constant (if more earthquake data are to be processed, constant is greater or equal to zero). Format (5A10, A6, 2I3, 2X, F5.1).

b. For each station (first S-waves, then P-waves):
   - station 3-letter code,
   - azimuth,
   - angle of incidence at the focus,
   - observed polarization angle or P-polarity (+1 or -1).

   Format (A3, F6.1, 2 F5.1).

c. - initial, final and incremental plunge of X-axis, measured from the vertical,
   - initial, final and incremental trend of X-axis,
   - initial, final and incremental trend of Y-axis,
   - constant (if more than one set of axes are to be tried, constant is greater than zero).

   Format (10I5).

2.6.2. <u>Output.</u>

The program prints out:

a. latitude, longitude, date, origin time and depth.

b. initial, final and incremental plunge of X-axis, trend of X-axis, trend of Y-axis, number of P-observations and number of S-observations.

c. for the values of x (= 0.01, 0.10, 0.50, 1.00, 5.00, 25.00 and 100.00):
   - plunge of the X-axis, measured from the vertical,
   - trend of X-axis,
   - trend of Y-axis,
   - number of inconsistent P-observations,
   - standard deviation of S-waves
   - and an error $E_{p\varepsilon}$

   x is a suitable multiplying factor and is equal to 1 if the reliability of both P- and S-wave observation is considered equal.

3. The choice of the program.

The question is now, which of the six programs is to be used in a particular case. It is clear that if only P observations are available, we can only use the programs of Kasahara and Wickens. For the program of Kasahara an approximation of the solution is needed. If this is not accessible we can only use the program of Wickens.

For a well-defined solution there must be a minimum of 100 P-observations. The use of S-wave polarization data makes it possible to produce a reliable mechanism solution with relatively fewer observations.

If P-observations are insufficient in number then a method, which combines P- and S-waves is used, or a method with only S-waves (Stevens or Hirasawa).

Where both P- and S-waves alone are insufficient we may use the programs of Baumann (Udias) or Chandra.

If sufficient P- and sufficient S-observations are available then each program can be used; in that case it is possible to compare the results.

The scheme below can be helpful to choose the best program.

	P observ.	S observ.	first approx.	execution time	max. observ.	computer language
Kasahara	yes	no	yes	low	190	FORTRAN
Wickens	yes	no	no	high	200	FORTRAN
Stevens	no	yes	no	moderate	50	FORTRAN
Hirasawa	no	yes	no?	low	60	ALGOL
Baumann	yes	yes	yes	high	100	FORTRAN
Chandra	yes	yes	yes	moderate	99	FORTRAN

The execution-time depends on the number of stations used, the accuracy of the assumed first approximation (Kasahara, Baumann/ Udias and Chandra), the required accuracy of the final approximation and the number of trial solutions (Hirasawa).

In the programs of Kasahara, Wickens, Baumann and Chandra weights are used for the P-waves.

Weights can be assigned to each station; in practice these weights would depend on the past reliability of the station.

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